Transformation matrix can transform the vector representation in one basis set into another.

$$\langle a_j | \underbrace{\hat{U} | a_i \rangle}_{|b_i\rangle} = \langle a_j | b_i \rangle$$

$\langle a_j | b_i \rangle$ → maxtrix element

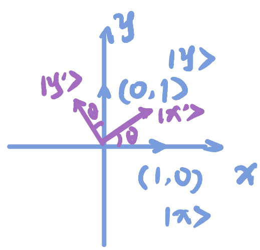
e.g., $\{ |x\rangle, |y\rangle \} \longrightarrow \{ |x'\rangle, |y'\rangle \}$

($|x\rangle = a_1$, $|y\rangle = a_2$, $|x'\rangle = b_1$, $|y'\rangle = b_2$)

$$\hat{R} = \begin{pmatrix} a_1 \cdot b_1 & a_1 \cdot b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix} = \begin{pmatrix} \langle x | x' \rangle & \langle x | y' \rangle \\ \langle y | x' \rangle & \langle y | y' \rangle \end{pmatrix}$$

↓

Rotation matrix in 2D ($\mathbb{R}$)



$|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|x'\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$ $|y'\rangle = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$

$$\hat{R} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

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eigenvalue, eigen vector:

$$\hat{x} |\alpha\rangle = a |\alpha\rangle$$

$$\mathbb{1} \hat{x} |\alpha\rangle = a \mathbb{1} |\alpha\rangle \longrightarrow (\hat{x} - a\mathbb{1}) |\alpha\rangle = \vec{0}$$

null ket if $|\alpha\rangle = \vec{0}$

$(\hat{x} - a\mathbb{1})$ has solution iff $(\hat{x} - a\mathbb{1})$ is not invertible.

$$\det(\hat{x} - a\mathbb{1}) = 0$$

*D 11 degeneracy:

if there are m linearly independent eigenkets of $\hat{x}$, that have the same eigenvalue a then the eigonvalue of these eigenkets $\{|a_i\rangle\}$ are said to be degenerate.

*T4: The eigenvalues of $\hat{U}$ are $u \in \mathbb{C}$ & $|u|=1$ and the eigenvectors of $\hat{U}$ are orthogonal.

$\rightarrow \hat{U}|u_i\rangle = u_i|u_i\rangle$ (assuming no denenracy)

$\rightarrow \langle u_j|\hat{U}^+\hat{U}|u_i\rangle = u_j^* u_i \langle u_j|u_i\rangle$

$\hat{U}^+\hat{U} \rightarrow 1$

$\rightarrow (u_j^* u_i - 1)\langle u_j|u_i\rangle = 0$ $\qquad u_i^* u_i = 1 \Rightarrow |u_i|^2 = 1$

if $i=j$ since $\langle u_i|u_i\rangle \neq 0 \Rightarrow u_j^* u_i = 1 \Rightarrow |u_i| = 1$ ($j = i$)

$i \neq j \Rightarrow |u_j\rangle \neq |u_i\rangle$ Because of no denenracy.

$u_i u_j^* \neq 1 \Leftarrow \{ u_j \neq u_i \Rightarrow u_j^* \neq u_i^* \Rightarrow u_i u_j^* \neq u_i u_i^* = 1$

Diagonalization & eigenvalue.

suppose $\{|a_i\rangle\}$ are linearly independent eigenkets. of $\hat{x} \Rightarrow \hat{x}|a_i\rangle = a_i|a_i\rangle$

then $\Rightarrow$

$$\hat{x}[|a_1\rangle \ |a_2\rangle \cdots] = [|a_1\rangle \ |a_2\rangle \cdots]\begin{bmatrix} a_1 & 0 & \cdots \\ 0 & a_2 & \\ \vdots & & \ddots \end{bmatrix}$$

$$[a_1|a_1\rangle \ a_2|a_2\rangle \cdots] = [a_1|a_1\rangle \ a_2|a_2\rangle \cdots]$$

Let $\hat{T} = [\,|a_1\rangle \ |a_2\rangle \cdots]$ Let $\hat{A} = \begin{pmatrix} a_1 & 0 & \cdots \\ 0 & a_2 & \\ \vdots & & \ddots \end{pmatrix}$

$\Rightarrow \hat{x}\cdot\hat{T} = \hat{T}\cdot\hat{A} \rightarrow \hat{T}^{-1}\hat{x}\hat{T} = \hat{A}$

→ if there exists $\hat{T}$ s.t. $\hat{T}^{-1}\hat{x}\hat{T} = \hat{A}$ then.

$\hat{x}$ is diagonalizable.

→ column vectors of $\hat{T}$ are eigenkets. of $\hat{x}$.

& the diag $[\because]$ contains the eigenvalues.

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2. Measurement, observable & uncertainty.

1> measurement.

P.A. Dirac: Measurement causes sys. to jump into an eigenstate of the dynamic variables that is being measured.

*. Before measuring. (of observable $\hat{A}$)

$$|\alpha\rangle = \sum_i c_i |a_i\rangle = \sum_i \overbrace{|a_i\rangle\langle a_i|}^{\hat{\Lambda}_i}\alpha\rangle$$

- c_i ($\in \mathbb{C}$)
- $\sum_i |a_i\rangle\langle a_i| \Rightarrow \mathbb{1}$
- $\langle a_i|\alpha\rangle \rightarrow c_i$
- $|\alpha\rangle$ → arbitrary vector living in the space spanned by the eigenkets of $\hat{A}$

*. measurement throws the sys. into one of the eigenstates.

$$|\alpha\rangle \xrightarrow[\text{measure}]{\hat{A}} |a_i\rangle$$

measurement changes the state. except for that the state is already in an eigenstate $|a_i\rangle \xrightarrow{\hat{A}} |a_i\rangle$

*. which $|a_i\rangle$ the sys will be thrown into is unknown. in advance.

However, the probability is known.

$$P_{a_i} = |\underbrace{\langle a_i|\alpha\rangle}_{\text{scalar} \in \mathbb{C}}|^2$$