$$\longrightarrow \left(\frac{\partial U}{\partial x}\right)_s = -\frac{\left(\frac{\partial S}{\partial x}\right)_U}{\underbrace{\left(\frac{\partial S}{\partial U}\right)_x}_{1/T}} = -T\underbrace{\left(\frac{\partial S}{\partial x}\right)_U}_{\Downarrow\, 0} = 0$$

B> $\left(\frac{\partial^2 U}{\partial x^2}\right) = ?\ (>0)$

Let $\left(\frac{\partial U}{\partial x}\right)_s \equiv P$, then $P = P(U, x)_s$

$$dP = \left(\frac{\partial P}{\partial U}\right)_x dU + \left(\frac{\partial P}{\partial x}\right)_U dx$$

$\Rightarrow$ divided by dx

has been proved to be "0"

$$\Rightarrow \left(\frac{\partial P}{\partial x}\right)_s = \underbrace{\left(\frac{\partial P}{\partial U}\right)_x \underbrace{\left(\frac{\partial U}{\partial x}\right)_s}}_{\to 0} + \left(\frac{\partial P}{\partial x}\right)_U$$

since $\left(\frac{\partial U}{\partial x}\right)_s \equiv P$

so. $\left(\frac{\partial P}{\partial x}\right)_s = \left(\frac{\partial^2 U}{\partial x^2}\right)_s$

$$\left(\frac{\partial P}{\partial x}\right)_U = \left(\frac{\partial}{\partial x}\left(-\frac{\left(\frac{\partial S}{\partial x}\right)_U}{\left(\frac{\partial S}{\partial U}\right)_x}\right)\right)$$

$P \to = \left(\frac{\partial U}{\partial x}\right)_s$

$$= \left(\frac{\partial^2 S}{\partial x^2}\right)\left(-\frac{1}{\left(\frac{\partial S}{\partial U}\right)_x}\right) + \underline{\left(\frac{\partial S}{\partial x}\right)_U \cdot \frac{\partial}{\partial x}\left(-\frac{1}{\left(\frac{\partial S}{\partial U}\right)_x}\right)}$$

$$\left.\frac{\partial}{\partial x}\left(-\frac{1}{\underbrace{\left(\frac{\partial S}{\partial U}\right)_x}_{\downarrow\, f(x)}}\right)\right\} \to g(x) = -\frac{1}{f(x)}\ \Big\} \to \frac{\partial g(x)}{\partial x} = \frac{\partial g(f(x))}{\partial x}$$

$$\rightarrow = \frac{\partial g}{\partial f}\cdot\frac{\partial f}{\partial x} \Rightarrow \frac{\partial}{\partial x}\left(-\frac{1}{\left(\frac{\partial S}{\partial U}\right)_x}\right) = \frac{1}{f^2(x)}\cdot\frac{\partial f}{\partial x}$$

$$= +\frac{1}{\left(\frac{\partial S}{\partial U}\right)_x^2}\cdot\frac{\partial}{\partial x}\left(\frac{\partial S}{\partial U}\right)_x$$

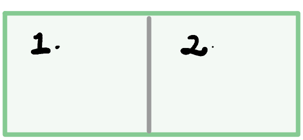
$$\left(\frac{\partial P}{\partial x}\right)_S \qquad P \equiv \left(\frac{\partial U}{\partial x}\right)_S$$

$$\Uparrow$$

$$\underbrace{\left(\frac{\partial P}{\partial x}\right)_U}_{\Downarrow\ \frac{\partial^2 U}{\partial x^2}} = \left(\frac{\partial^2 S}{\partial x^2}\right)\left(-\frac{1}{\underbrace{\left(\frac{\partial S}{\partial U}\right)_x}_{\downarrow\ \frac{1}{T}}}\right) + \underbrace{\left(\frac{\partial S}{\partial x}\right)_U}_{\Downarrow\ 0}\cdot\cancel{\left(+\frac{1}{\left(\frac{\partial S}{\partial U}\right)_x^2}\cdot\frac{\partial}{\partial x}\left(\frac{\partial S}{\partial U}\right)_x\right)}$$

$$= \underbrace{\left(\frac{\partial^2 S}{\partial x^2}\right)}_{\Downarrow\ <0}\cdot\underbrace{(-T)}_{\Downarrow\ <0} > 0$$

An example.



→ isolated.

↓

rigid, diathermal

(impermeable)

Equilibrium Condition

based U min principle?

U_{min}.

$$\rightarrow dU = 0 = dU_1 + dU_2$$

$$= đQ_1 + đQ_2$$

$$= T_1 dS_1 + T_2 dS_2$$

$$S_{total} = S_1 + S_2 = \text{const.}$$

$$\Rightarrow S_2 = \text{const.} - S_1$$

$$\Rightarrow dS_2 = -dS_1 \quad \Rightarrow dU = 0 = T_1 dS_1 - T_2 dS_1$$

$$= dS_1 \underbrace{(T_1 - T_2)}_{\Downarrow \; T_1 = T_2} = 0$$

---

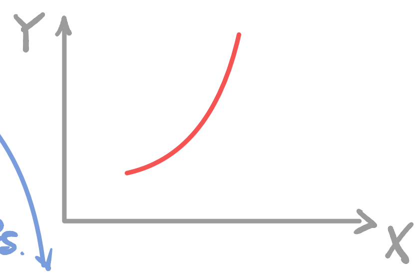
2. Legendre Transformation.

Try to involve Intensive Parameters.

However, thermodynamic info is incomplete.

e.g., Fundamental Eq. $Y = Y(X)$

Intensive parameter. $P \equiv \dfrac{\partial Y}{\partial X}$



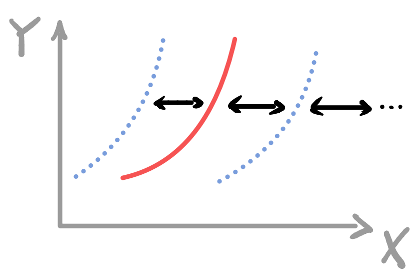
if we use $Y = Y(P)$

can we go back to $Y = Y(X)$

→ No, we will have arbitrary shifts.

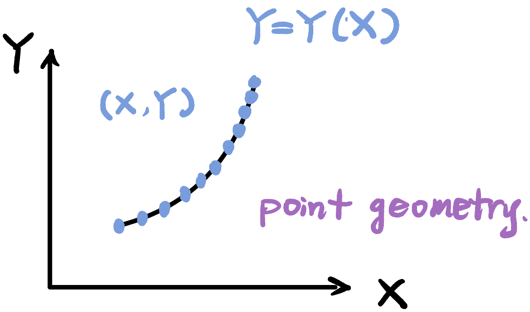
→ incomplete info.

(But "P" is more experimentally relevant.)

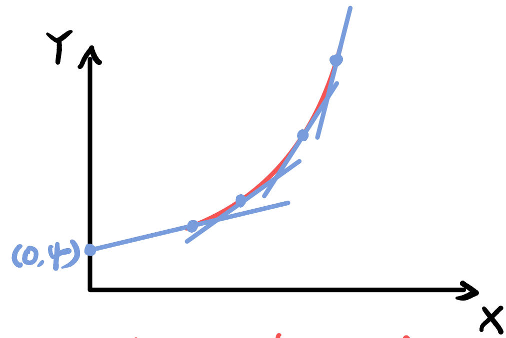


*. How to avoid such info. loss. (while have "P")

⇒ Point Geometry ⇔ line geometry.



1> locus of points (X, Y) satisfying $Y = Y(X)$



2> the envelope of a family of tangent lines w/ slope of "P" and intersect of "ψ", satisfying $\psi = \psi(P)$

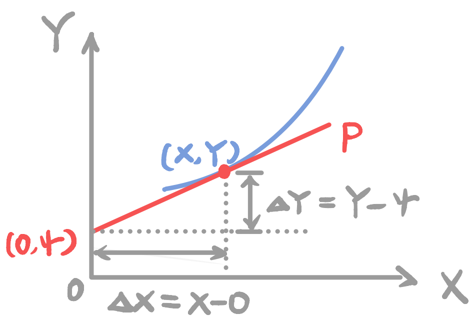
$$\Rightarrow \quad Y = Y(X) \Leftrightarrow \psi = \psi(P)$$

↓

involving intensive parameter $\{(P, \psi)\}$

$$Y = Y(X) \overset{?}{\Leftrightarrow} \psi = \psi(P)$$

what's the relation between (P, ψ) & (X, Y)



$$P = \frac{\partial Y}{\partial X} \quad \left(= \frac{\widetilde{\Delta Y}}{\widetilde{\Delta X}}\right)$$

↓

$$P = P(X)$$

↓

$$\boxed{X = X(P)}$$

$$\Rightarrow P = \frac{Y-\psi}{X-0} \Rightarrow \psi = Y - PX$$

$$\psi = Y(X) - P(X)\cdot X = Y(X(P)) - P(X(P))\cdot X(P)$$

---

conversely. if we know $\psi = \psi(P)$ $(= Y - PX)$

$$d\psi = dY - d(PX) = dY - P\,dx - x\,dP$$

$$P \downarrow \frac{\partial Y}{\partial x}$$

$$= \cancel{dY} - \cancel{\frac{\partial Y}{\partial x}}\cancel{dx} - x\,dP = -x\,dP$$

$$\Rightarrow -x = \frac{d\psi}{dP} \rightarrow x = x(P) \rightarrow \boxed{P = P(x)}$$

eliminate "P"

$$Y = \psi - PX = \psi(P) - P\cdot X(P)$$

$$= \psi(P(X)) - P(X)\cdot X(P(X))$$