Obj. 1 Essence of QM.

1. Kets, bras. & operators (change of basis)
2. Measurements, observables. and uncertainty.
3. Position Momentum and Translation.
4. Wavefunction in position and momentum space.
5. Spin ½ system.

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1. Kets, bras. & operators (change of basis)

1> Ket. $\underline{|\alpha\rangle}$

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ket vector (state vector)

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complete info about the physical state.

$c|\alpha\rangle \sim |\alpha\rangle \to$ same physical state.

key definitions & thereoms.

$\{|i\rangle\}$

* D1: Linear independence: $\sum_{i=0}^{n} a_i |i\rangle = 0$

Linear dependence = if NOT iff $a_i = 0 \ \forall a_i$

*.D2: Basis: a set of "n" linearly independent vectors in an n-D space.

*. T1: if $\{|i\rangle, i=1,2,\cdots n\}$ is linearly indep. in n-D $\rightarrow$ $\underline{|\alpha\rangle = \sum_{i=1}^{n} a_i |i\rangle}$ $\forall\ \alpha$ in n-D

Expansion by a basis set.

2> Bras space & inner product.

$\langle\alpha|$ ~ $\underbrace{\text{mirror image}}_{\text{"dual"}}$ of $|\alpha\rangle$ ket.

$|\alpha\rangle \doteq \psi_\alpha$

$$\langle\alpha| \xleftrightarrow{DC} |\alpha\rangle \rightarrow \begin{bmatrix} a_1 \\ a_2 \\ \vdots \end{bmatrix}$$

$\langle\alpha| \rightarrow \psi_\alpha^*$ $\qquad$ $|\alpha\rangle \rightarrow \psi_\alpha$

$\langle\alpha| \rightarrow [a_1^*\ a_2^* \cdots]$ $\qquad$ $(a_i \in \mathbb{C})$

$|\alpha\rangle \rightarrow$ vector space V

$\langle\alpha| \rightarrow$ Dual vector space V^*

$\downarrow$ eat the vectors in V and produce a scalar.

$\Rightarrow$ $\underline{\langle\beta|\alpha\rangle} \rightarrow$ scalar.

$\downarrow$

inner product. $\longleftrightarrow$ $\int dx\ \psi^*(x)\phi(x)$ $\updownarrow$ $\langle\psi|\phi\rangle$

Axioms:

A1: conjugate symmetry.

$$\langle\alpha|\beta\rangle = (\langle\beta|\alpha\rangle)^*$$

$$(\langle\beta|\alpha\rangle)^* = \left([b_1^*\ b_2^* \cdots]\begin{bmatrix} a_1 \\ a_2 \\ \vdots \end{bmatrix}\right)^* = (b_1^* a_1 + b_2^* a_2 + \cdots)^* = (b_1 a_1^* + b_2 a_2^* + \cdots) = [a_1^*\ a_2^* \cdots]\begin{bmatrix} b_1 \\ b_2 \\ \vdots \end{bmatrix} = \langle\alpha|\beta\rangle$$

A2: Positive Semidefinite:

$$\underbrace{\langle\alpha|\alpha\rangle}_{\text{must be real}} \geq 0 \longrightarrow \text{essentially modulus square. } \| |\alpha\rangle \|^2$$

A3: Linearity (in ket)

$$\langle\beta|(a_1\alpha_1 + a_2\alpha_2)\rangle = a_1\langle\beta|\alpha_1\rangle + a_2\langle\beta|\alpha_2\rangle$$

* Anti-Linearity (in bras)

$$\langle(b_1\beta_1 + b_2\beta_2)|\alpha\rangle = \langle b_1\beta_1|\alpha\rangle + \langle b_2\beta_2|\alpha\rangle$$

$$= b_1^*\langle\beta_1|\alpha\rangle + b_2^*\langle\beta_2|\alpha\rangle$$

*D3 orthogonality: $\leftrightarrow \langle\beta|\alpha\rangle = 0$

*D4. Norm (or vector length): $\sqrt{\langle\alpha|\alpha\rangle} \equiv \| |\alpha\rangle \|$

Normalize ket $|\tilde{\alpha}\rangle = \dfrac{|\alpha\rangle}{\sqrt{\langle\alpha|\alpha\rangle}}$

*D5: Orthonormal basis.

$\begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \end{bmatrix}$ basis set $\{|i\rangle\}$ $\begin{cases} ① \text{ unit norm.} \\ ② \text{ orthogonal (pairwise)} \end{cases}$

$\rightarrow |\alpha\rangle = \sum_i^n a_i|i\rangle \leftrightarrow \begin{bmatrix} a_1 \\ a_2 \\ \vdots \end{bmatrix}$ ① $= 1$ if $j = i$; ② $= 0$ if $j \neq i$

$\rightarrow |\alpha\rangle = \sum_i^n a_i|i\rangle$

$|\beta\rangle = \sum_j^n b_j|j\rangle \Rightarrow \langle\beta| = \sum_j^n b_j^*\langle j|$

$\Rightarrow \langle\beta|\alpha\rangle = \sum_j\sum_i \overbrace{b_j^* a_i \langle j|i\rangle}$

Kronecker Delta $\delta_{ji} \begin{cases} = 1 & j = i \\ = 0 & j \neq i \end{cases}$