

## Obj. 1 Essence of QM.

1. Kets, bras. & operators (change of basis)
  2. Measurements, observables, and uncertainty.
  3. Position Momentum and Translation.
  4. Wavefunction in position and momentum space.
  5. Spin  $\frac{1}{2}$  system.
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### 1. Kets, bras. & operators (change of basis)

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$\Rightarrow$  Ket:  $\frac{|\alpha\rangle}{\downarrow}$   
Ket vector (state vector)  
 $\downarrow$   
complete info about the physical state.

$c|\alpha\rangle \sim |\alpha\rangle \rightarrow$  same physical state.

### key definitions & theorems.

{1}

\* D1: Linear independence:  $\sum_{i=0}^n a_i |i\rangle = 0$

Linear dependence = if NOT iff  $a_i = 0 \forall a_i$

\* D2: Basis: a set of "n" linearly independent vectors in an n-D space.

\*. TL: if  $\{|i\rangle, i=1, 2, \dots, n\}$  is linearly indep.

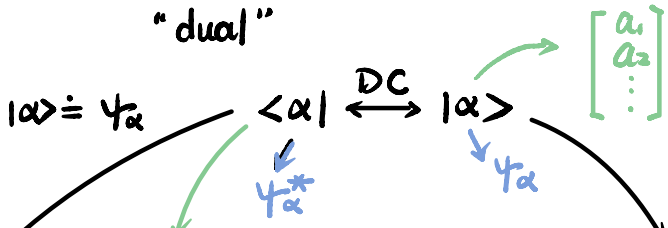
in  $n$ -D  $\rightarrow$   $|\alpha\rangle = \sum_{i=1}^n a_i |i\rangle \neq \alpha$  in  $n$ -D

Expansion by a basis set.

2> Bras space & inner product.

$\langle\alpha|$  - mirror image of  $|\alpha\rangle$  ket.

"dual"



$|\alpha\rangle \doteq \psi_\alpha$

$[a_1^* \ a_2^* \ \dots]$      $(a_i \in \mathbb{C})$

vector space  $V$

Dual vector space  $V^*$

eat the vectors in  $V$  and produce a scalar.

$\Rightarrow \langle\beta|\alpha\rangle \rightarrow$  scalar.

inner product.  $\longleftrightarrow \int dx \psi^*(x)\phi(x)$

$\hookrightarrow \langle\psi|\phi\rangle$

Axioms:

A1: conjugate symmetry.

$([b_1^* \ b_2^* \ \dots] \begin{bmatrix} a_1 \\ a_2 \\ \vdots \end{bmatrix})^* = (b_1^* a_1 + b_2^* a_2 + \dots)^*$

$\langle\alpha|\beta\rangle = (\langle\beta|\alpha\rangle)^* = (b_1 a_1^* + b_2 a_2^* + \dots)$

$= [a_1^* \ a_2^* \ \dots] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \end{bmatrix} = \langle\alpha|\beta\rangle$

A2: Positive Semidefinite:

$$\frac{\langle \alpha | \alpha \rangle \geq 0}{\text{must be real}} \rightarrow \text{essentially modulus square.}$$

$\|\alpha\|^2$

A3: Linearity (in ket)

$$\langle \beta | (a_1 \alpha_1 + a_2 \alpha_2) \rangle = a_1 \langle \beta | \alpha_1 \rangle + a_2 \langle \beta | \alpha_2 \rangle$$

\* Anti-Linearity (in bras)

$$\begin{aligned} \langle (b_1 \beta_1 + b_2 \beta_2) | \alpha \rangle &= \langle b_1 \beta_1 | \alpha \rangle + \langle b_2 \beta_2 | \alpha \rangle \\ &= b_1^* \langle \beta_1 | \alpha \rangle + b_2^* \langle \beta_2 | \alpha \rangle \end{aligned}$$

\*D3 orthogonality:  $\leftrightarrow \langle \beta | \alpha \rangle = 0$

\*D4. Norm (or vector length):  $\sqrt{\langle \alpha | \alpha \rangle} \equiv \|\alpha\|$

$$\text{Normalize ket } |\tilde{\alpha}\rangle = \frac{|\alpha\rangle}{\sqrt{\langle \alpha | \alpha \rangle}}$$

\*D5: Orthonormal basis.

$$\begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} \text{ basis set } \{|i\rangle\} \begin{cases} \textcircled{1} \text{ unit norm.} \\ \textcircled{2} \text{ orthogonal (pairwise)} \end{cases}$$

$$\rightarrow |\alpha\rangle = \sum_i^n a_i |i\rangle \leftrightarrow \begin{bmatrix} a_1 \\ a_2 \\ \vdots \end{bmatrix} \begin{cases} \textcircled{1} = 1 \text{ if } j=i \\ \textcircled{2} = 0 \text{ if } j \neq i \end{cases}$$

$$\begin{aligned} \rightarrow |\alpha\rangle &= \sum_i^n a_i |i\rangle \\ |\beta\rangle &= \sum_j^n b_j |j\rangle \Rightarrow \langle \beta | \alpha \rangle = \sum_j \sum_i \overbrace{b_j^* a_i} \langle j | i \rangle \\ &\Rightarrow \langle \beta | = \sum_j^n b_j^* \langle j | \quad \text{Kronecker Delta } \delta_{ji} \begin{cases} = 1 & j=i \\ = 0 & j \neq i \end{cases} \end{aligned}$$