$$\Rightarrow \frac{d^2 \hat{x}_i}{dt^2} = \frac{d}{dt}\left(\frac{\hat{p}_i}{m}\right) = -\frac{1}{m}\left(-\frac{\partial V(\hat{x})}{\partial \hat{x}_i}\right)$$

For Gradient $\nabla f(x)$

$$x = (x_1, x_2, x_3, \cdots)$$

$$= \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix} = \frac{\partial f}{\partial x_1}\vec{x}_1 + \frac{\partial f}{\partial x_2}\vec{x}_2 + \cdots \frac{\partial f}{\partial x_n}\vec{x}_n$$

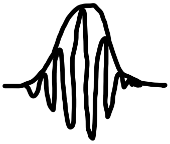
$$\frac{d^2 \hat{x}}{dt^2} = \begin{pmatrix} \frac{d^2 \hat{x}_i}{dt^2} \\ \frac{d^2 \hat{x}_j}{dt^2} \\ \frac{d^2 \hat{x}_k}{dt^2} \end{pmatrix} = \frac{-1}{m}\begin{pmatrix} \frac{\partial V(\hat{x})}{\partial \hat{x}_i} \\ \frac{\partial V(\hat{x})}{\partial \hat{x}_j} \\ \frac{\partial V(\hat{x})}{\partial \hat{x}_k} \end{pmatrix} = -\frac{1}{m}\nabla V(\hat{x})$$

$$\Rightarrow \boxed{\frac{d^2 \hat{x}}{dt^2} = -\frac{1}{m}\nabla V(\hat{x})}$$

$$\boxed{m\frac{d^2}{dt^2}\langle \hat{x} \rangle = \frac{d\langle \hat{p} \rangle}{dt} = -\langle \nabla V(\hat{x}) \rangle}$$

Ehrenfest theorem

$\hbar$ disappears : center of wave packet moves like classical particle subject to $V(\hat{x})$.



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*. 2-5. state ket & Base ket.

state ket : $|\alpha, t_0\rangle$

Base ket : $|a'\rangle$

$$\Rightarrow |\alpha, t_0\rangle = \sum_{a'} C_{a'} |a'\rangle = \sum_{a'} |a'\rangle\langle a'|\alpha, t_0\rangle$$

$$\boxed{\hat{A}^{(S)} |a'\rangle^{(S)} = a' |a'\rangle^{(S)}}$$

$$\hat{A}^{(H)} = \hat{u}^{+} \hat{A}^{(S)} \hat{u}$$

$$\Rightarrow \underbrace{\hat{u}^{+} \hat{A}^{(S)} \hat{u}}_{\hat{A}^{(H)}} \hat{u}^{+} |a'\rangle^{(S)} = \hat{u}^{+} a' |a'\rangle^{(S)}$$

$$\Rightarrow \hat{A}^{(H)} \left(\hat{u}^{+} |a'\rangle^{(S)} \right) = a' \left(\hat{u}^{+} |a'\rangle^{(S)} \right)$$

$(\hat{u}^{+}|a'\rangle)$ is Heisenberg base ket.

$\Rightarrow$ (H): $|a', t\rangle = \hat{u}^{+} |a'\rangle^{(S)}$, $\hat{u}^{+} = \hat{u}^{-1}$

(S): $|\alpha, t_0; t\rangle = \hat{u} |\alpha, t_0\rangle$

$\Rightarrow |\alpha\rangle^{(H)}$ is static

$\Rightarrow$ Base ket in (H) are "rotating" oppositely compared to the state ket in (S).

$\Rightarrow |a', t\rangle^{(H)}$ satisfies the (S) eq. But w/ the wrong sign.

$$\rightarrow \boxed{i\hbar \frac{\partial}{\partial t} |a', t\rangle^{(H)} = \underset{*}{-} \hat{H}^{(S)} |a', t\rangle^{(H)}}$$ How?

*. consistency

$$C_{a'}(t) = \underbrace{\langle a'|}_{(S)} \underbrace{(\hat{u}|\alpha, t_0\rangle)}_{(S)}$$

$$\parallel$$

$$C_{a'}(t) = (\langle a'|\hat{u})|\alpha, t_0\rangle$$

$$= \underbrace{\langle \hat{u}^{+} a'|}_{(H)} \underbrace{\alpha, t_0\rangle}_{(H)}$$

	schrödinger	Heisenberg
$\|\alpha\rangle$	"moving"	stationary
$\hat{A}$	(generally) stationary	moving
$\|a'\rangle$	stationary	moving (But w/ $\hat{u}^{+}$)

---

3. Harmonic Oscillator

3-1. Energy eigenkets & eigenvalues.

$$\hat{H} = \frac{\hat{P}^2}{2m} + V(\hat{x}) = \frac{\hat{P}^2}{2m} + \frac{m w^2 x^2}{2}$$

$$w = \sqrt{\frac{k}{m}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = m w^2$$

k ↓ stiffness of spring.

$$U(x) = -\int F dx = -\int (-kx) dx = \frac{1}{2} k x^2$$

$\hat{x}, \hat{p}, \hat{H} \rightarrow$ Hermitian.

*Def 1. Annihilation operator.

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i\hat{p}}{m\omega}\right)$$

Creation operator

$$\hat{a}^{+} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i\hat{p}}{m\omega}\right)$$

$\Downarrow$

$\hat{a} \neq \hat{a}^{+} \Rightarrow$ non-Hermitian.

$$\Rightarrow \boxed{[\hat{a}, \hat{a}^{+}] = \mathbb{1}}\ *$$

*. Def 2. Number operator. $\hat{N} = \hat{a}^{+}\hat{a}$

$$\Rightarrow \hat{N} = \frac{\hat{H}}{\hbar\omega} - \frac{1}{2}$$

$$\Rightarrow \hat{H} = \hbar\omega\left(\hat{N} + \frac{1}{2}\right) = \hbar\omega\left(\hat{a}^{+}\hat{a} + \frac{1}{2}\right)$$

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why they're named as creation & annihilation operators?

Denote eigenket & eigenvalue of $\hat{N}$

$$\hat{N}|\lambda\rangle = \lambda|\lambda\rangle \qquad (\langle\lambda|\lambda\rangle = 1)$$

$$\Rightarrow \hat{H}|\lambda\rangle = \underline{\hbar\omega\left(\lambda + \frac{1}{2}\right)}|\lambda\rangle$$

Energy eigenvalue $E_\lambda = \hbar\omega\left(\lambda + \frac{1}{2}\right)$

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commutator $\underbrace{[\hat{N}, \hat{a}]}_{\Downarrow\, -\hat{a}}$ & $\underbrace{[\hat{N}, \hat{a}^{+}]}_{\Downarrow\, \hat{a}^{+}}$
