$$t = \frac{(-Q_h)}{\sigma_h (T_h - T_w)} + \frac{Q_c}{\sigma_c (T_t - T_c)}$$

(Assume the adiabatic process takes neglible time:)

$$W = \varepsilon \cdot (-Q_h) = \left(1 - \frac{\tilde{T}_c}{\tilde{T}_h}\right)(-Q_h) = \frac{T_w - T_t}{T_w}(-Q_h)$$

$\tilde{T}_c \to T_t$, $\tilde{T}_h \to T_w$

$$\rightarrow \quad (-Q_h) = \frac{T_w}{T_w - T_t} \cdot W$$

$$Q_c = \frac{\tilde{T}_c}{\tilde{T}_h}(-Q_h) = \frac{T_t}{T_w}(-Q_h) = \frac{T_t}{T_w - T_t} \cdot W$$

$$\Rightarrow \quad \underline{t} = \frac{(-Q_h)}{\sigma_h (T_h - T_w)} + \frac{Q_c}{\sigma_c (T_t - T_c)}$$

$$= \frac{1}{\sigma_h (T_h - T_w)} \cdot \frac{T_w}{T_w - T_t} \cdot \underline{W} + \frac{1}{\sigma_c (T_t - T_c)} \cdot \frac{T_t}{T_w - T_t} \cdot \underline{W}$$

$$\frac{W}{t} = \left(\frac{1}{\sigma_h (T_h - T_w)} \cdot \frac{T_w}{\underset{?}{\underline{T_w}} - \underset{?}{\underline{T_t}}} + \frac{1}{\sigma_c (T_t - T_c)} \cdot \frac{T_t}{T_w - T_t} \right)^{-1}$$

optimized values, such that $\frac{W}{t}$ is max

$\left(\frac{W}{t}\right)_{max}$ is given by

$$\begin{cases} \underline{T_w = C \cdot (T_h)^{1/2}} \\ \underline{T_t = C \cdot (T_c)^{1/2}} \end{cases}$$

$$C = \frac{(\sigma_h T_h)^{1/2} + (\sigma_c T_c)^{1/2}}{\sigma_h^{1/2} + \sigma_c^{1/2}}$$

$$\rightarrow \left(\frac{W}{t}\right)_{max} = \sigma_h \sigma_c \left[\frac{T_h^{1/2} - T_c^{1/2}}{\sigma_h^{1/2} + \sigma_c^{1/2}} \right]$$

$$\rightarrow \varepsilon_E = 1 - \frac{T_t}{T_w} = 1 - \left(\frac{T_c}{T_h}\right)^{1/2}$$

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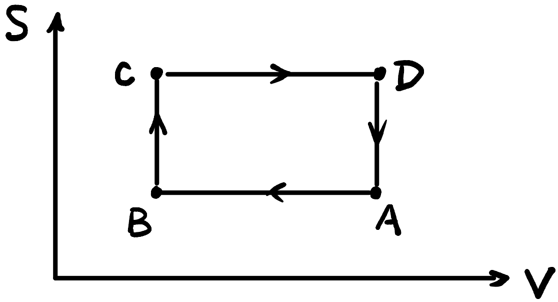
5. Other engine cycles.

1> Otto engine.



A→B: Adiabatic Compression

B→C: Heat at const. V.
(isochoric)

C→D: Adiabatic Expansion.

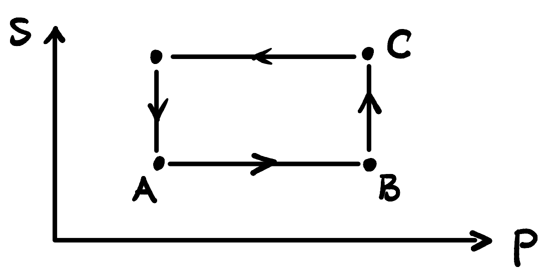
D→A: cooling at const. V.

Efficiency: $\varepsilon_{otto} = 1 - \left(\frac{V_B}{V_A}\right)^{\frac{C_p - C_v}{C_v}}$ (For ideal gas)

$$\varepsilon_{otto} = \underbrace{\frac{Q_{Hot} - Q_{cold}}{Q_{Hot}}}_{\text{General}} = 1 - \frac{Q_C}{Q_H} = 1 - \underbrace{\frac{\cancel{C_v}(T_D - T_A)}{\cancel{C_v}(T_C - T_B)}}_{\text{Otto cycle.}}$$

How to convert $T \rightarrow V$?

2> Joule Cycle.



A→B: Adiabatic Compression.

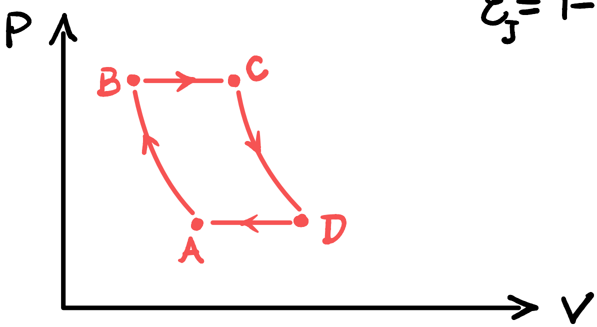
B→C: Isobaric combustion.

C→D: Expansion, & Exhaust to atmosphere.
(Isentropic)

D→A: Fresh air taken into engine.

For ideal gas.

$$\varepsilon_J = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{C_P - C_V}{C_P}}$$



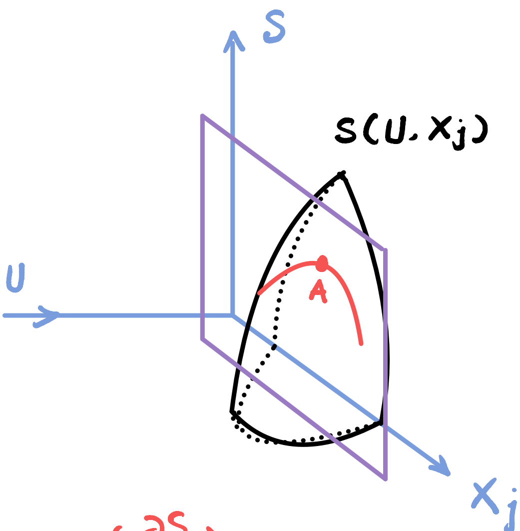
Obj. 7 Legendre Transformation.

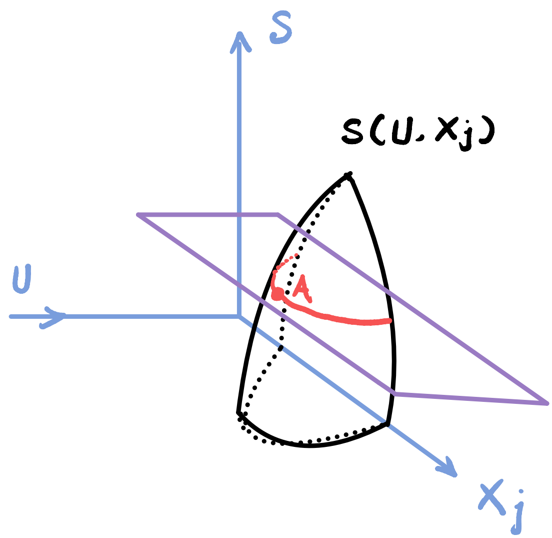
1. Energy minimum principle.
2. Legendre Transformation.
3. Thermodynamic potentials (e.g., H, G, F, ⋯)
4. Massieu Functions.

1. Energy minimum principle.

$\Updownarrow$

Postulate II: Entropy max principle.





* $\left(\frac{\partial S}{\partial U}\right) > 0$

* U is single-valued continuous function of S.

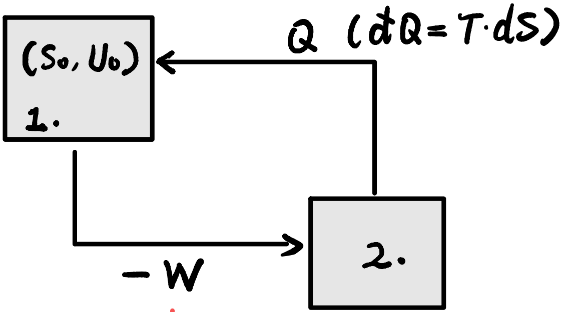
Energy minimum principle.

"A" point is Energy minimized.

why? they're equivalent?

1> Intuitively

what if for a given sys. the initial S_0, U_0, U_0 is not minimized.



Adiabatically withdrawing work.

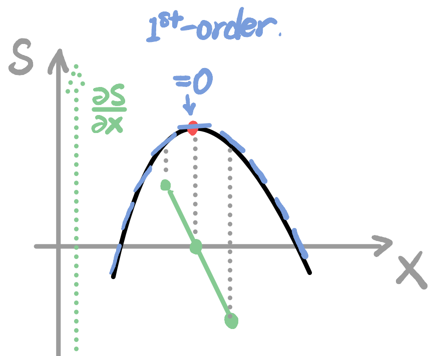
2> Prove (math)

Assuming Entropy max

① Max → Extremum.

$$\rightarrow \left(\frac{\partial S}{\partial x}\right) = 0$$

② Max → $\left(\frac{\partial^2 S}{\partial x^2}\right) < 0$



$$\frac{\partial}{\partial x}\left(\frac{\partial S}{\partial x}\right) < 0$$

$$\underbrace{\frac{\partial}{\partial x}}_{x}\underbrace{\left(\frac{\partial S}{\partial x}\right)}_{y}$$

?

Prove 1> $\left(\frac{\partial U}{\partial x}\right) = 0$ 2> $\left(\frac{\partial^2 U}{\partial x^2}\right) > 0$

A> $\underline{(U, X, S)}$

$$\downarrow$$

$$\left(\frac{\partial U}{\partial x}\right)_S \left(\frac{\partial x}{\partial S}\right)_U \left(\frac{\partial S}{\partial U}\right)_x = -1$$

$$\longrightarrow \left(\frac{\partial U}{\partial x}\right)_S = - \frac{\left(\frac{\partial S}{\partial x}\right)_U}{\underbrace{\left(\frac{\partial S}{\partial U}\right)_x}_{1/T}} = -T \underbrace{\left(\frac{\partial S}{\partial x}\right)_U}_{\Downarrow\; 0} = 0$$

B> $\left(\frac{\partial^2 U}{\partial x^2}\right) = ?$ (>0)