$$|\alpha\rangle \xrightarrow[\text{measure}]{\hat{A}} |a_i\rangle$$

measurement changes the state. except for that the state is already in an eigenstate $|a_i\rangle \xrightarrow{\hat{A}} |a_i\rangle$

*. which $|a_i\rangle$ the sys will be thrown into.

is unknown. in advance.

However, the probability is known.

$$P_{a_i} = |\underbrace{\langle a_i|\alpha\rangle}_{\text{scalar} \in \mathbb{C}}|^2$$

Fundamental postulate in QM.

*. D12. Expectation value of $\hat{A}$ w/ respect to state $|\alpha\rangle$

$$\langle \hat{A} \rangle \equiv \langle \alpha | \hat{A} | \alpha \rangle$$

$$\hat{A}|a_i\rangle = a_i |a_i\rangle$$

$$\langle \hat{A} \rangle_\alpha = \sum_i \sum_j \underbrace{\langle \alpha | a_j \rangle}_{\langle a_j|\alpha\rangle^* = c_j^*} \langle a_j | \hat{A} | a_i \rangle \underbrace{\langle a_i | \alpha \rangle}_{c_i}$$

$\hat{\Lambda}_j$ $\hat{\Lambda}_i$ $\mathbb{1}$

$$= \langle a_j | a_i | a_i \rangle$$

$$= a_i \langle a_j | a_i \rangle = \begin{cases} j \neq i \to 0 \\ j = i \end{cases}$$

$$\sum_i a_i \underbrace{|\langle a_i|\alpha\rangle|^2}_{\text{Probability for obtaining } a_i}$$

2>-1. compatible observables.

*D13 compatible $[\hat{A}, \hat{B}]=0$

incompatible $[\hat{A}, \hat{B}] \neq 0$

---

* $[\hat{A}, \hat{B}]=0 \rightarrow$?

① $\underline{\hat{A}|a_i\rangle = a_i|a_i\rangle}$

$\rightarrow \hat{A}(\underline{\hat{B}|a_i\rangle}) = \hat{B}(\hat{A}|a_i\rangle) = \hat{B}(a_i|a_i\rangle)$

$= a_i(\underline{\hat{B}|a_i\rangle})$

$\Rightarrow \hat{B}|a_i\rangle$ is an eigenket of $\hat{A}$ ($\hat{A}$ has non-degenerate eigenvalues)

$\Rightarrow (\hat{B}|a_i\rangle)$ & $|a_i\rangle$ should not be different by more than a multiplicative

$\Rightarrow \hat{B}|a_i\rangle = b_i|a_i\rangle$ ← $[\hat{A}, \hat{B}]=0$

$\Rightarrow$ $\boxed{\hat{A} \text{ \& } \hat{B} \text{ have common eigenbasis.}}$

$\Leftrightarrow \hat{A}$ & $\hat{B}$ are compatible observables.

② $\hat{A}$ & $\hat{B}$ have common eigenbasis.

$$\hat{A}\hat{B}|\alpha\rangle = \hat{A}\hat{B}\sum_i c_i|a_i\rangle = \hat{A}\sum_i c_i\hat{B}|a_i\rangle$$

$c_i = \langle a_i|\alpha\rangle$

$$= \hat{A}\sum_i c_i b_i|a_i\rangle$$

$$= \sum_i c_i b_i a_i|a_i\rangle$$

likewise.

$$\hat{B}\hat{A}|\alpha\rangle = \cdots \sum_i c_i a_i b_i|a_i\rangle$$

$$\Rightarrow \quad \hat{A}\hat{B} = \hat{B}\hat{A} \Rightarrow [\hat{A},\hat{B}] = 0$$

③ Degenerate case (true)

if $[\hat{A},\hat{B}] = 0$ & $\hat{A}|a_{i,j}\rangle = a_i|a_{i,j}\rangle$

$j = 1, 2, \cdots n.$

$\{|a_{i,j}\rangle\}$ linearly independent.

$\hat{A}$ & $\hat{B}$ still have common eigenbasis.

---

compatible

$$|\alpha\rangle \xrightarrow[\text{measure}]{\hat{A}\ \nearrow a_i} |a_i\rangle \xrightarrow{\hat{B}\ \nearrow b_i} |a_i\rangle \xrightarrow{\hat{A}\ \nearrow a_i} |a_i\rangle$$

$$|\alpha\rangle = \sum_i c_i|a_i\rangle$$

$\hat{A}|a_i\rangle = a_i|a_i\rangle$

$\hat{B}|a_i\rangle = b_i|a_i\rangle$

→ subsequent measurements do not destroy the info obtained in the previous measurement.

2>-2. incompatible.

$[\hat{A},\hat{B}]\neq 0 \Leftrightarrow \hat{A}$ & $\hat{B}$ do not have a complete set of silmultaneous eigenkets

Proof: if they do

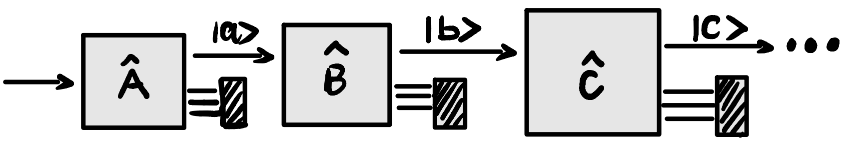
$$\Rightarrow \hat{A}\hat{B}|a_i\rangle = \hat{A}\, b_i|a_i\rangle = b_i a_i|a_i\rangle$$

likewise, $\hat{B}\hat{A}|a_i\rangle = \hat{B}\, a_i|a_i\rangle = a_i b_i|a_i\rangle$

$\Rightarrow \hat{A}\hat{B} = \hat{B}\hat{A} \Rightarrow [\hat{A},\hat{B}]=0$ controdicting $[\hat{A},\hat{B}]\neq 0$

---

①



$|\langle c|b\rangle|^2\,|\langle b|a\rangle|^2 \rightarrow$ Probability of obtaining $|c\rangle$

total probability through all "b" channels.

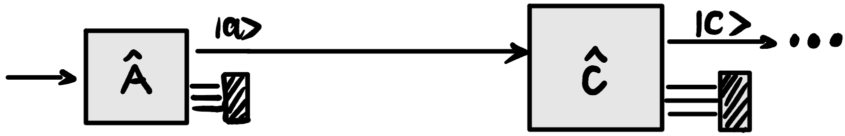
$$\Rightarrow \sum_b |\langle c|b\rangle|^2\,|\langle b|a\rangle|^2$$

$$= \sum_b \langle c|b\rangle\langle c|b\rangle^*\,\langle b|a\rangle\langle b|a\rangle^*$$

$$= \sum_b \langle c|b\rangle\langle b|c\rangle\langle b|a\rangle\langle a|b\rangle$$

①

2



$$P_c = |\langle c|a\rangle|^2 = |\sum_b \langle c\underbrace{|b\rangle\langle b|}_{\hat{\Lambda}_b}a\rangle|^2$$

$$= \left(\sum_b \langle c|b\rangle\langle b|a\rangle\right)\left(\sum_{b'} \langle c|b'\rangle\langle b'|a\rangle\right)^*$$

$$= \sum_b \sum_{b'} \langle c|b\rangle\langle b|a\rangle\langle a|b'\rangle\langle b'|c\rangle$$

2

$$\hat{A}|a\rangle = a|a\rangle \quad \hat{B}|b\rangle = b|b\rangle \quad \hat{C}|c\rangle = c|c\rangle$$

$$= \sum_b \langle c|b\rangle\langle b|a\rangle \sum_{b'} \langle a|b'\rangle\langle b'|c\rangle$$

if $[\hat{A}, \hat{B}] = 0$, $\Rightarrow$ then $|a\rangle, |b\rangle$ are simultaneous eigenkets

$$\left(|a\rangle, |b\rangle \in \underbrace{\{|a_i\rangle\}}_{\hat{A}\ \&\ \hat{B}} \right)$$

$\Rightarrow$ only one of $|b\rangle$s will be good for $\langle b|a\rangle \neq 0$

$$\Rightarrow = \sum_b \langle c|b\rangle\langle b|a\rangle\langle a|b\rangle\langle b|c\rangle$$

3

if $[\hat{A}, \hat{B}] = 0$; $|a\rangle$ will not be "damaged" after coming out from $\hat{B}$, so, it's the same w/ or w/o $\hat{B}$.

on the other hand, if $[\hat{A}, \hat{B}] \neq 0$.
results from $\hat{C}$ depends on whether or not
$\hat{B}$ measurement has actually been performed.

3.> **Uncertainty:**